sharkydevil

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Consider the miquel point of quadrilateral $BCFE$, this is, the intersection of circles (AEF) and (ABC), we say this point is the A-sharkydevil point of $\triangle ABC$

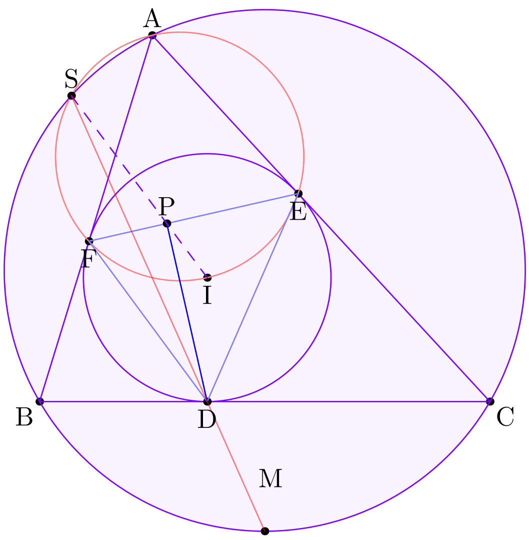

Figure 1: The A-Sharkydevil point.

This point have many nice propierties, we are going to present the more known in the following table

Teorema 0.1 (Most known propierties in Skarkydevil)

Using Figure 1 notation

1) SD bisects $\angle BSC$ then it passes through the midpoint M of arc $\widehat{BC}$
2) P be the foot from D to EF then S is the inverse of P wrt incircle
3) AS and BC meet on AI perpendicular through I
4) SD and (AEF) meets on $A-$altitude

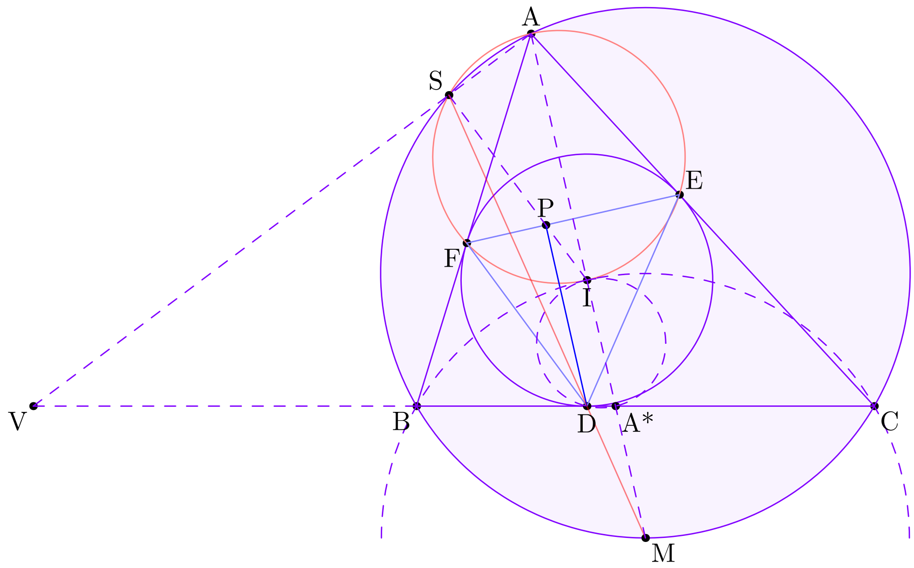

Figure 2: A-Sharkydevil propierties

Claim 0.2 — SD bisects $\angle BSC$ then it passes through the midpoint M of arc $\widehat{BC}$

Proof. invert wrt (BIC), A goes to $A^* = BC \cap AM$, (AEF) goes to (A^*ID) then so S goes to D. □

Claim 0.3 — P be the foot from D to EF then S is the inverse of P wrt incircle

Proof. invert wrt incircle, (AEF) goes to EF, as this inversion swaps (ABC) to the nine-point circle of $\triangle DEF$, then P is the inverse of S □

Claim 0.4 — AS and BC meet on AI perpendicular through I

Proof. use Radical Axis Theorem on $(AFE), (BIC)$ and (ABC) □

Claim 0.5 — SD and (AEF) meets on $A-$altitude

Proof. Invert wrt (BIC), let $R = (AEF) \cap SD$, $R^* = (IDA^*) \cap SD$ so

$$\angle MID = \angle MR^*A^* = \angle MAR$$

consider O the center of (ABC), then O goes to $O^* = MO \cap (BIC)$ then $BOCM$ is a rhombus and note that

$$\angle MAO = \angle MO^*A^* = \angle O^*MA^* = \angle MID = \angle MAR$$

ergo AU and AO are isogonal □

Ejemplo 0.6 (Mexico TST 2024)

Let Ω be the circumcircle of $\triangle ABC$ with incenter I. Let $M \neq A$ be the intersection of AI and Ω, and $D \in BC$ such that $ID \perp BC$. Let $E \in \Omega$ such that $AE \perp BC$. Let $N \neq I$ the intersection of ID and (BIC). Prove that NE and MD intersect at Ω

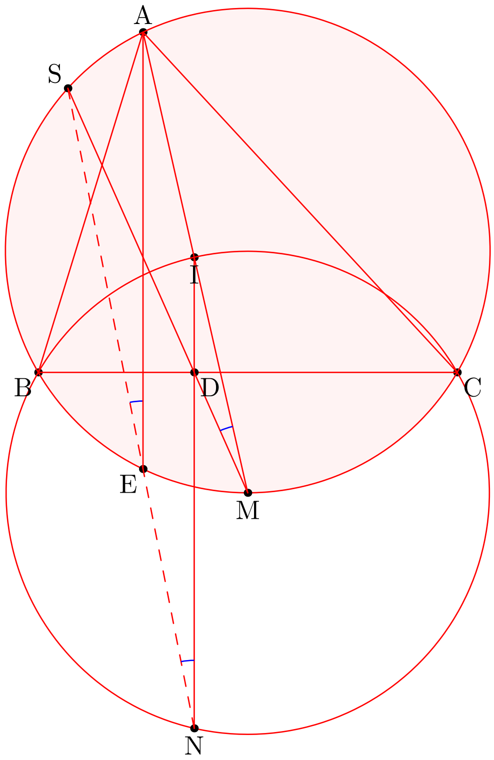

Figure 3: Mexico TST/2024

We instantly note that $S = MD \cap (ABC)$ is the $A-$sharkydevil point, now note that

$$\angle AMS = \angle AES = \angle INS$$

as $IS \parallel AE$ we are done; the last inequality follows by $IMNS$ being cyclic which is true because the inversion wrt (BIC) send $\overline{IDN}$ to $IMNS$